

# Entanglement distillation using particle statistics

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We extend the idea of entanglement concentration for pure states(Phys. Rev. Lett. **88**, 187903) to the case of mixed states. The scheme works only with particle statistics and local operations, without the need of any other interactions. We show that the maximally entangled state can be distilled out when the initial state is pure, otherwise the entanglement of the final state is less than one. The distillation efficiency is a product of the diagonal elements of the initial state, it takes the maximum 50%, the same as the case for pure states.

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Entanglement shared between two distinguishable particles is generally an advantage for quantum information processing, while indistinguishability prevents us from addressing the particles separately that seems to be a disadvantage in information processing. It is only recently that researchers have started to investigate the use of particle statistics (both bosonic and fermionic) for quantum information processing[1, 2, 3]. It was shown that quantum statistics can lead to an effective interaction between internal and external degrees of freedom of particles[1], and then result in space-spin entanglement transfer. The other useful tasks such as entanglement concentration[2] and state discrimination[3]could also be accomplished, even if nonoptimally, using only the effects of quantum statistics, without the need of any other interactions. These proposals purely based on particle statistics differ significantly from some previous suggestions, where particle statistics together with interparticle interactions [4, 5, 6] were used for quantum information processing.

The entanglement distillation/concentration is essential in quantum communication and computation, it increases the entanglement shared between distant parties, and consequently could better the performances in quantum information processing. As entanglement can not be increased by local operations and classical communication[7], the distillation/concentration operation is to distill/concentrate entanglement locally from a large to a smaller number of pairs[8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

Quantum information processing based on particle statistics is very useful for tasks implemented with identical particles. In this paper, we extend the protocol proposed in Ref.[2] for pure states to mixed states. To start with, we recall and summarize the entanglement concentration protocol using only particle statistics as follows. The initial state is prepared at two sides labelled by  $L$  and  $R$ . In each side, two parties Alice (A) and Bob (B) share  $n$  pairs of particles. The protocol includes the following four steps. (1) Bring each of Bob's particles (from both sides of  $L$  and  $R$ ) into a 50/50 beam splitter; (2) Before the particles from the side  $L$  fall into the beam splitter, a specific unitary transformation that flips the particle's spin is applied to each of the particles; (3)

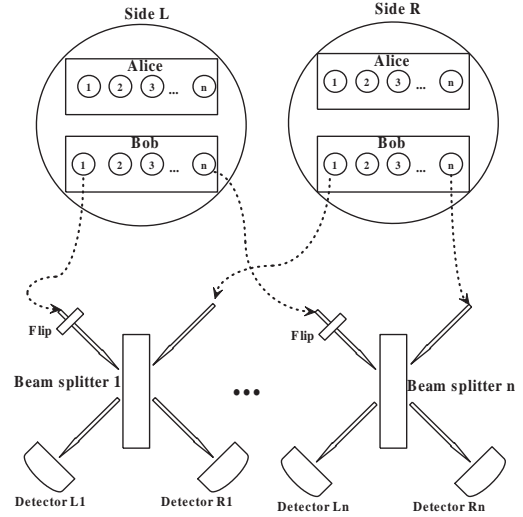


FIG. 1: An schematic illustration of the scheme. Alice and Bob start with two pairs of entangled systems at side  $L$  and  $R$ , respectively. At each side, Alice and Bob share  $n$  pairs of particles. After a set of local operations including spin flip and 50/50 beam splitter, path measurements, and classical communications, states with more entanglement can be distilled out from the initial state. The scheme works with particle statistics solely, without the need of other interactions.

Make a specific measurement on the output particles, and discard those which bunch(for fermions)/antibunch(for bosons); (4) Repeat the above three steps for all particles with Bob. This was schematically illustrated in figure 1. We now apply the above steps to mixed states for fermions, the results can be straightforwardly generalized to the case of bosons. As the authors did in Ref.[2], we consider the entanglement in the internal degrees of freedom of the particles, for example the spin in the case of fermions. In either sides  $L$  or  $R$ , Alice and Bob share  $n$  pairs of particles, the initial state at side  $I$  ( $I = R, L$ ), distributed between Alice and Bob, in basis of  $\{|A\uparrow\rangle_I^n|B\downarrow\rangle_I^n, |A\downarrow\rangle_I^n|B\uparrow\rangle_I^n\}$  is

$$\rho_I^n = \begin{pmatrix} a & c \\ c^* & b \end{pmatrix}. \quad (1)$$

where  $|A\tilde{\uparrow}\rangle_L^n = |\underbrace{\uparrow\uparrow\ldots\uparrow}_n\rangle_{L,A}$  denotes  $n$  spin-up particles at  $L$  side with Alice. The other notations in the basis are similar to  $|A\tilde{\uparrow}\rangle_L^n$ . The total initial state under consideration is then

$$\rho^n = \rho_L^n \otimes \rho_R^n = \begin{pmatrix} a^2 & ac & ac & c^2 \\ ac^* & ab & |c|^2 & bc \\ ac^* & |c|^2 & ab & bc \\ c^{*2} & bc^* & bc^* & b^2 \end{pmatrix}. \quad (2)$$

Here, the basis chosen is

$$\begin{aligned} |\alpha\rangle &\equiv |A\tilde{\uparrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B\tilde{\downarrow}\rangle_L^n |B\tilde{\downarrow}\rangle_R^n, \\ |\beta\rangle &\equiv |A\tilde{\uparrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B\tilde{\downarrow}\rangle_L^n |B\tilde{\uparrow}\rangle_R^n, \\ |\gamma\rangle &\equiv |A\tilde{\downarrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B\tilde{\uparrow}\rangle_L^n |B\tilde{\downarrow}\rangle_R^n, \\ |\kappa\rangle &\equiv |A\tilde{\downarrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B\tilde{\uparrow}\rangle_L^n |B\tilde{\uparrow}\rangle_R^n. \end{aligned} \quad (3)$$

With this total initial state, next step we let the first pair of particles (labelled by 1 in the circle) with Bob go to the beam splitter 1, after having the specific unitary transformation in step (2), the total state still takes the form of Eq.(2), but was written in basis of  $\{|A\tilde{\uparrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B\tilde{\uparrow}\rangle_L^n |B\tilde{\downarrow}\rangle_R^n, |A\tilde{\uparrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B\tilde{\uparrow}\rangle_L^n |B\tilde{\uparrow}\rangle_R^n, |A\tilde{\downarrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B\tilde{\downarrow}\rangle_L^n |B\tilde{\downarrow}\rangle_R^n, |A\tilde{\downarrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B\tilde{\downarrow}\rangle_L^n |B\tilde{\uparrow}\rangle_R^n\}$ . Similarly, the state of particles after the first pair having passed through the 50/50 beam splitter can be written in the same form as in Eq.(2) in the Hilbert space spanned by

$$\begin{aligned} |\alpha_p\rangle &\equiv |A\tilde{\uparrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B_1\rangle_{L1R1} |B\tilde{\uparrow}\rangle_L^{n-1} |B\tilde{\downarrow}\rangle_R^{n-1}, \\ |\beta_p\rangle &\equiv |A\tilde{\uparrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B\tilde{\uparrow}\rangle_L^n |B\tilde{\uparrow}\rangle_R^n, \\ |\gamma_p\rangle &\equiv |A\tilde{\downarrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B\tilde{\downarrow}\rangle_L^n |B\tilde{\downarrow}\rangle_R^n, \\ |\kappa_p\rangle &\equiv |A\tilde{\downarrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B_2\rangle_{L1R1} |B\tilde{\downarrow}\rangle_L^{n-1} |B\tilde{\uparrow}\rangle_R^{n-1}, \end{aligned} \quad (4)$$

where  $|B_1\rangle_{L1R1} = \frac{1}{2}i(|B\uparrow\rangle_{L1}|B\downarrow\rangle_{L1} + |B\uparrow\rangle_{R1}|B\downarrow\rangle_{R1}) + \frac{1}{2}(|B\uparrow\rangle_{L1}|B\downarrow\rangle_{R1} + |B\downarrow\rangle_{L1}|B\uparrow\rangle_{R1})$ , and  $|B_2\rangle_{L1R1} = -\frac{1}{2}i(|B\uparrow\rangle_{L1}|B\downarrow\rangle_{L1} + |B\uparrow\rangle_{R1}|B\downarrow\rangle_{R1}) + \frac{1}{2}(|B\downarrow\rangle_{L1}|B\uparrow\rangle_{R1} + |B\uparrow\rangle_{L1}|B\downarrow\rangle_{R1})$ . When the particle had passed through the beam splitter, Bob performs a path measurement on the first pair with assumption that the detectors do not absorb the particles and do not disturb their internal (spin) degrees of freedom. Discarding those particles which bunch, we arrive at

$$\rho_1^n = N_1^2 \begin{pmatrix} \frac{1}{2}a^2 & \frac{1}{\sqrt{2}}ac & \frac{1}{\sqrt{2}}ac & \frac{1}{2}c^2 \\ \frac{1}{\sqrt{2}}ac^* & ab & |c|^2 & \frac{1}{\sqrt{2}}bc \\ \frac{1}{\sqrt{2}}ac^* & |c|^2 & ab & \frac{1}{\sqrt{2}}bc \\ \frac{1}{2}c^{*2} & \frac{1}{\sqrt{2}}bc^* & \frac{1}{\sqrt{2}}bc^* & \frac{1}{2}b^2 \end{pmatrix}, \quad (5)$$

where  $N_1 = (\frac{1}{2}a^2 + \frac{1}{2}b^2 + 2ab)^{-1/2}$ , and the corresponding basis is the same as that in Eq.(4), but the bunching terms in  $|B_1\rangle_{L1R1}$  and  $|B_2\rangle_{L1R1}$  were discarded. The probability of having state Eq.(5) is  $(0.5 + ab)$ , it arrives

at the maximum  $3/4$  with  $a = b = 1/2$ . Eq.(4) tells us that the bunching is only related to  $|\alpha_p\rangle$  and  $|\kappa_p\rangle$ , and these two terms have probability  $1/2$  of antibunching. Thus elements of the density matrix Eq.(5) differ from the initial state upon the normalization factor  $1/\sqrt{2}$  or  $1/2$ , depending on the total probability of antibunching. With the antibunching state as an initial state, next we let the second pair of Bob's particles (labelled by 2 in the circle) pass through another 50/50 beam splitter and perform the same measurement, keeping again the antibunching results. Having completed the step (4) and absorbed all these  $(n-1)$  particles, we get the final state

$$\rho_n^n = N_n^2 \begin{pmatrix} (\frac{1}{2})^n a^2 & (\frac{1}{\sqrt{2}})^n ac & (\frac{1}{\sqrt{2}})^n ac & (\frac{1}{2})^n c^2 \\ (\frac{1}{\sqrt{2}})^n ac^* & ab & |c|^2 & (\frac{1}{\sqrt{2}})^n bc \\ (\frac{1}{\sqrt{2}})^n ac^* & |c|^2 & ab & (\frac{1}{\sqrt{2}})^n bc \\ (\frac{1}{2})^n c^{*2} & (\frac{1}{\sqrt{2}})^n bc^* & (\frac{1}{\sqrt{2}})^n bc^* & (\frac{1}{2})^n b^2 \end{pmatrix} \quad (6)$$

with  $N_n = ((\frac{1}{2})^n a^2 + (\frac{1}{2})^n b^2 + 2ab)^{-1/2}$ , and the basis

$$\begin{aligned} |\alpha_f\rangle &\equiv |A\tilde{\uparrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B_{triplet}\rangle, \\ |\beta_f\rangle &\equiv |A\tilde{\uparrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B\tilde{\uparrow}\rangle_L |B\tilde{\uparrow}\rangle_R, \\ |\gamma_f\rangle &\equiv |A\tilde{\downarrow}\rangle_L^n |A\tilde{\uparrow}\rangle_R^n |B\tilde{\downarrow}\rangle_L |B\tilde{\downarrow}\rangle_R, \\ |\kappa_f\rangle &\equiv |A\tilde{\downarrow}\rangle_L^n |A\tilde{\downarrow}\rangle_R^n |B_{triplet}\rangle. \end{aligned} \quad (7)$$

Here,  $|B_{triplet}\rangle = \frac{1}{\sqrt{2}}(|B\uparrow\rangle_{L1}|B\downarrow\rangle_{R1} + |B\downarrow\rangle_{L1}|B\uparrow\rangle_{R1})$ . The probability of having this state is  $p_f = ((\frac{1}{2})^n a^2 + (\frac{1}{2})^n b^2 + 2ab)$ . With  $n \rightarrow \infty$ , the probability  $p_f = 2ab$ , while the entanglement measured by Wootters concurrence is  $|c|^2/ab$ . The entanglement of the final state reaches its maximal value 1 with  $|c|^2 = ab$ , i.e., in the pure state case. For mixed states, because  $|c|^2 < ab$  required by Eq.(1), the entanglement of the final state is always less than unity, and it is of relevance to the coherence of state  $\rho_f^n$  measured by  $|c|$  in Eq.(1). On the other hand, the state Eq.(1) itself is an entangled state, the entanglement measured by Wootters concurrence is  $2|c|$ . To get more entanglement in the final state, it is required that  $|c| > 2ab$ . Entangled state with  $|c| \leq 2ab$  could not be distilled, this is similar to the non-distillable entanglement called bound entanglement.

In conclusion, we have extended the entanglement concentration protocol to the case of mixed states, which uses only the effects of particle statistics. The maximal entanglement after distillation depends on the off-diagonal elements of the initial state that usually is a measure of coherence. The distillation efficiency only depends on the diagonal element of the initial state, it takes 50% for finite number of particles and tends to 25% for infinite large states.

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